Glasgow Counts

A Framework for Mathematics

Approaches to Learning and Teaching Numeracy at Primary Level

Effective Learning and Teaching in Mathematics

Mission Statement

Our fundamental aim is to fill our young minds with a sense of agency and endow them with the motivation, courage and belief in their power to influence their own futures. We are driven by a commitment to create pathways to enable all stakeholders to possess skills for life, learning and work.

We want our young people to engage with mathematics and build their comprehension of the subject across the curriculum.

Glasgow Counts
A Framework for Mathematics

Society requires young people who are sophisticated mathematical thinkers, pattern spotters and problem solvers therefore we aim to empower our young people as mathematicians. With this pathway we aim to provide opportunities for learning that promote deep engagement with all areas of mathematics.

Our purpose is to offer a better way to build mathematical understanding in and beyond our classrooms. **Glasgow Counts**.

Aims

This framework aims to support development of the two key areas of mathematical subject knowledge.

These are:

Mathematical knowledge

Through CPD establishments will be supported in developing a deeper understanding of mathematics.

Pedagogical knowledge

The framework draws on models and images to develop conceptual understanding of particular mathematical ideas.

Distinctive features

Progression

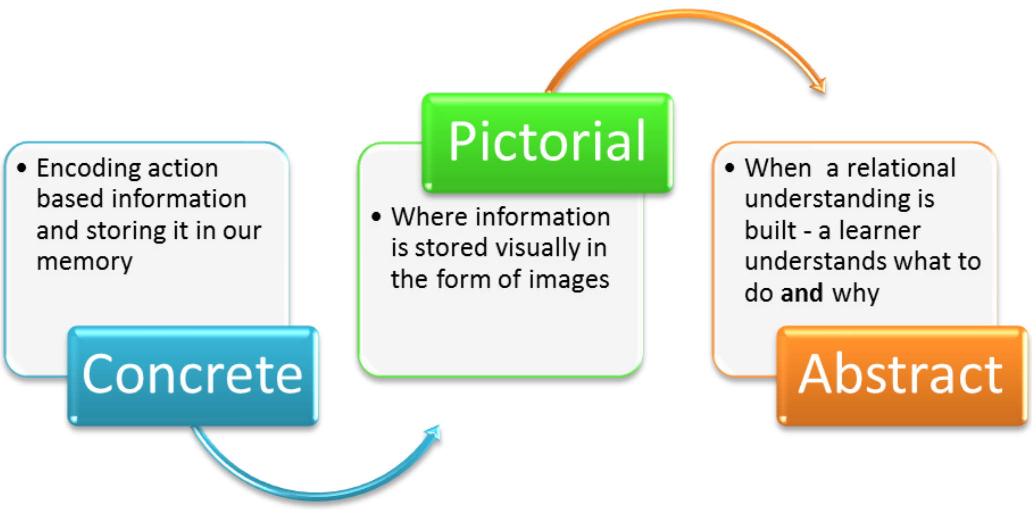
The learning progressions within each strand are incremental. This will support establishments to ensure that all learners reach their full potential.

Big Ideas

These sections deal with the ideas which underpin each particular strand of mathematics covered. This allows establishments to see the big picture immediately and understand how the different strands knit together.

Models and images

These sections deal with the best models and images to represent the elements of mathematics in each strand. This will help establishments in choosing appropriate representations when planning mathematics lessons.



Problem Solving, Reasoning and Fluency

The CFE supports a problem solving approach and promotes the development of children's problem solving, reasoning and fluency skills. If the three areas outlined are embedded in learning and teaching there will be a range of positive outcomes, including the development of children's conceptual understanding, their ability to use maths in meaningful ways and positive attitudes from the Early Level to Third Level.

Problem Solving

This can be summarised as the ability to apply mathematics to a variety of situations (Cockcroft, 1982) and Glasgow Counts encourages the use of 'low threshold, high ceiling' activities. These mathematical activities are designed so that the all learners can access and extend at their own level. They can lead to the development of a community of practice, positive attitudes and progression through deepening subject knowledge, rather than accelerating through content.

There are a wealth of activities on the NRich website (www.nrich.maths.org.) which can be used alongside the CFE.

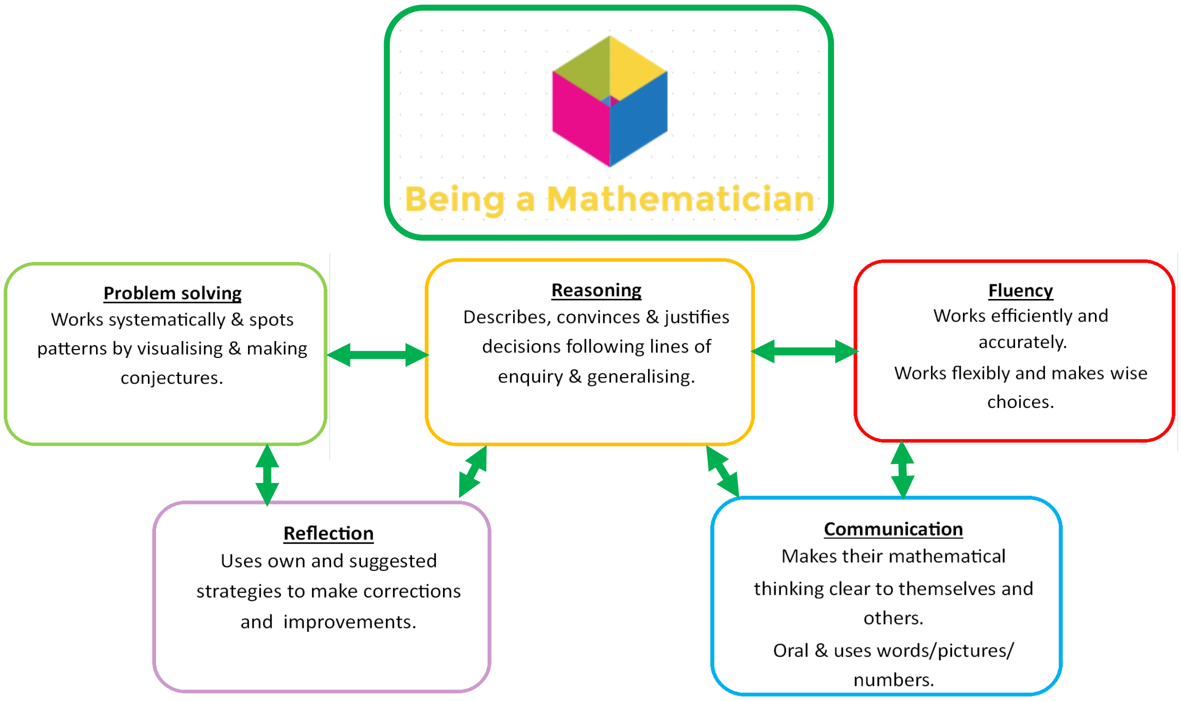
Reasoning

Reasoning can be considered the glue which holds maths together. A focus on the mathematical process and a commitment to children's understanding, as distinct from any final product, enables the development of reasoning skills in the primary classroom. By focusing on specific questioning techniques within mathematics lessons encourages learners to move from describing to explaining to justifying. It is another tool with which to challenge the higher attaining children.

Fluency

Developing learners mathematical fluency demands a focus on their efficiency, accuracy and flexibility. It requires them to know why they are doing what they are doing, and to make appropriate choices (from a toolkit of mental calculation strategies, for example). By using manipulatives (as part of a CPA approach), encouraging children to discuss their work, particularly through reasoning, and consolidating understanding across a range of meaningful contexts, children's fluency skills will develop.

These three aims are interrelated and complementary. They inform Glasgow Counts and are deemed to be at the root of high quality maths teaching and learning.



Cockcroft, W H (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*, London, Her Majesty's Stationery Office
McClure, L (ND) *Using Low Threshold High Ceiling Tasks in Ordinary Classrooms*, available online at http://nrich.maths.org/7701

Concrete → Pictorial → Abstract

WHAT?

CPA (Concrete, Pictorial, Abstract) is an approach to teaching mathematics based on the work of Jerome Bruner (1960). Bruner's premise was that children's conceptual understanding develops from being actively engaged in their learning and making sequential process through three stages of representation: enactive, iconic and symbolic (mapped onto **c**oncrete, **p**ictorial, **a**bstract respectively). Each stage builds on the previous one, although unlike Piagetian theory, they are not age-related.

CPA therefore encompasses multiple models that approach a concept at different cognitive levels. Firstly at the concrete level, children are exposed to a range of appropriate manipulatives, for example, dienes, unifix, Numicon, tens frames, straws, dot patterns, counters, shapes, coins and dice. Use of these concrete objects engages children with their learning and can provide a 'hook' into the learning. Another advantage of this approach is that discussion is a natural by-product of active learning which is an element of good quality maths teaching and learning (Williams, 2008).

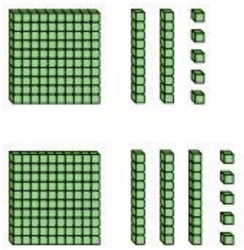
Progress into the pictorial phase is consequently underpinned by active, memorable experiences leading to deep learning. This second phase aids visualisation and the bar model is a key element of the pictorial phase of problem solving (this is explored later).

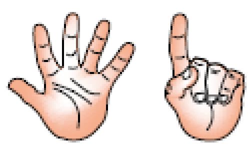
It is important to note that although the ultimate aim of a CPA strategy is to culminate in a fluent, abstract approach characterised by quick, efficient methods, the process should not be rushed. It may be necessary to return to previous phases to address children's misconceptions and consolidate their conceptual understanding.

Another key feature of the CPA process is that although concrete objects may be perceived as too elementary for upper primary learners (Sousa, 2007), both concrete and pictorial representations should be used across the primary phase.

WHY?

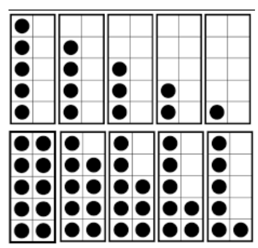
A commitment to CPA is intrinsic to effective learning and teaching in mathematics. It informs pedagogy and planning and is a supportive way of developing children's deep conceptual understanding, clear progression and positive attitudes to maths.

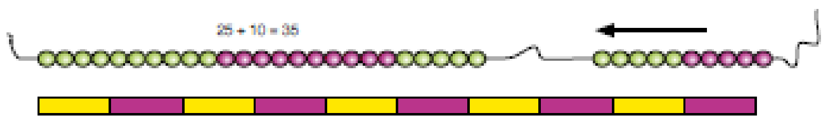


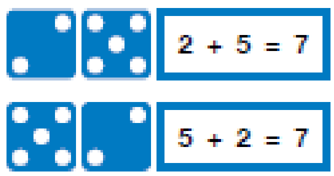


How?

Within this framework and through core CPD sessions, explanations and examples of appropriate CPA approaches, models and images are consistently discussed. Each strand has clear diagrams demonstrating the progression from the concrete to pictorial to abstract. This structure informs teachers' planning and pedagogy, as does reflection on best practice.







Addition and Subtraction

The underlying pattern of additive reasoning is the relationships between the parts and the whole. If children think and talk about the whole and parts using concrete materials early on then solid foundations should be laid and therefore learners can internalise this underlying pattern. Every time learners think and talk about addition and subtraction, they can be practising identifying the whole, breaking it into parts and then recombining to make the whole once more.

1-1 Correspondence

Is the ability to match each member of one set to a member of an equal set. Learners need to count objects in rows and in a collection to ensure they fully comprehend 1-1 correspondence.

Subitising

Subitising is an ability to instantaneously recognise the number of objects in a small group without the need to count all. It is an essential part of developing number sense in early years by helping them to relate numbers to actual items or groups of items.

Part—whole models

The ability to see a whole number as made of a number of different parts builds on the notion of subitising. By using the model for number bonds learners begin to understand subtraction as the undoing of addition and vice versa.

Number Lines

The number line acts as a visual representation of the process carried out in addition and subtraction. Learners will at first need to use the marked number line before moving to an empty or open number line to visualise their calculation.

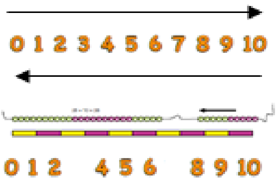
NB It is important to note that although the models and images suggested have been presented in a progressive form; learners do not learn in a linear fashion. **"We learned ...by doing, through direct experience, through dealing with things as they arose, and through discovering what it was that was important at the time. But most of all, we learned through making connections between stuff we already knew and the stuff we didn't. This meant we actively constructed the knowledge as we needed it. It was all very subjective and individual and not linear."**[1] This means that at any point learners may move forwards and backwards between the representations therefore it is essential that every learner has access to concrete materials to support the conceptual development of additive reasoning.

1 http://ken-carroll.com/2007/12/13/linear-and-non-linear-learning/ ; Ken Caroll, December 2007

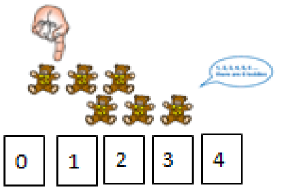
Addition and Subtraction

Building blocks...

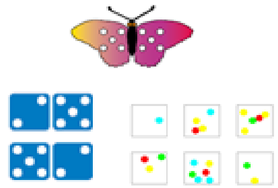
Number word sequences. Forward and back.



Counting with 1-1 correspondence.

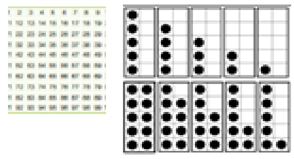


Subitising



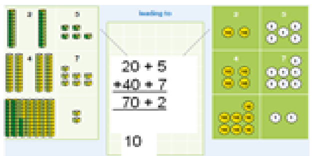
Finding one more and combining two groups Taking away with real objects

Counting in ones and tens Counting back in tens

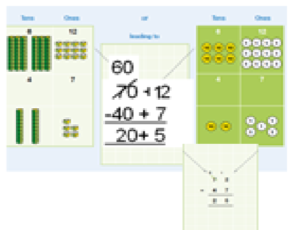


Leading to...

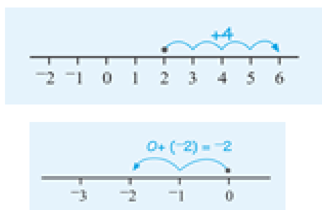
Expanded method in columns using appropriate resources



Compact column method for addition and subtraction

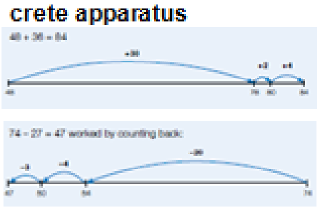


Using a marked number line, using symbols

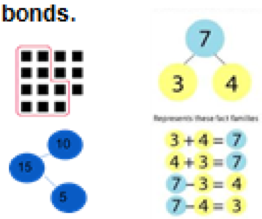


Finding the difference

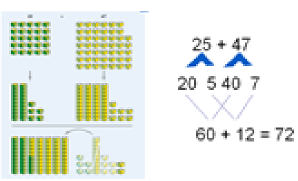
Blank /open number lines for addition relating to concrete apparatus



Addition and subtraction strategies and number bonds.



Partitioning for addition and subtraction using concrete resources



I can analyse a problem and apply the most effective strategy (for me) in the circumstances.

- Expanded column addition and subtraction
- Using a marked number line to compensate or transform
- Using an open number line for addition and subtraction
- Partition and recombine
- Round and adjust and number bonds
- Find the difference
- Column method

Multiplication and Division

Multiplicative reasoning is the ability to work flexibly between the concepts of multiplication and division. The transition from additive reasoning to multiplicative reasoning is a shift from understanding the relationship between the parts and the whole, to the understanding of grouping. Learner's misconceptions with multiplicative reasoning is often the result of memorised procedures without any concrete understanding of the models and strategies used to inform them.

Counting in groups

Is the ability to count in any amount. Learners need to be able to assign a value to a group and count on to find the total. The use of concrete resources and visual apparatus such as the counting stick will be useful in mastering this skill.

Arrays

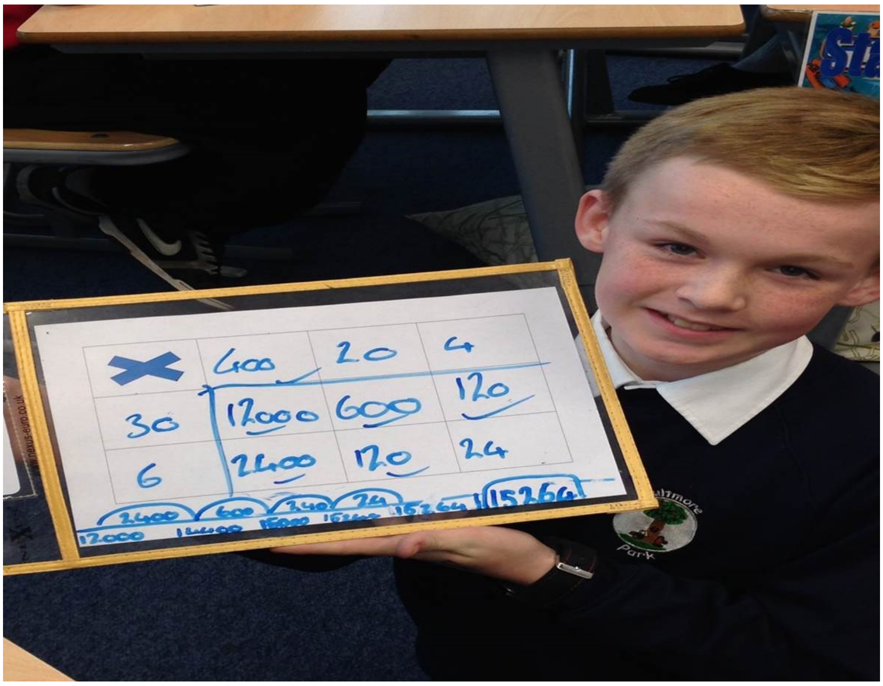
Julia Anghileri (2009) tells us that 'Children's first experiences of multiplication arise when they make groups with equal numbers of objects and recognise the possibility of counting the groups rather than counting individual **items.'** The use of arrays builds on this notion and allows learners to link multiplication and division.

Grouping and sharing

It is important that learners experience sharing and grouping. Grouping can be more easily equated to multiplication as in 5 'groups of' 2 equals 10 and 10 can be grouped in 5s or 2s. Learners will undoubtedly understand the concept of a 'fair share' however it is also important to explore 'unequal sharing' to build an understanding of remainders in a context.

Non Standard Methods

The use of jottings, number lines, arrays, repeated addition and subtraction and grid method will enable learners to visualise the process of multiplication and division. Learners should utilise these methods over standard written form and should be encouraged to use them as a prompt for articulating their thinking.



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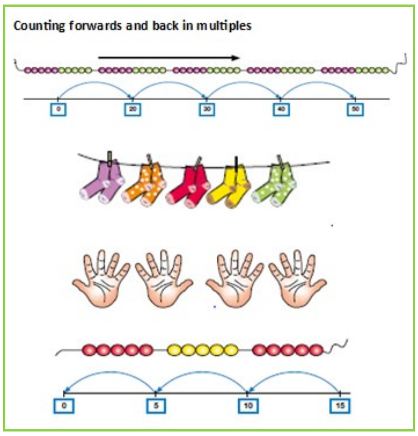
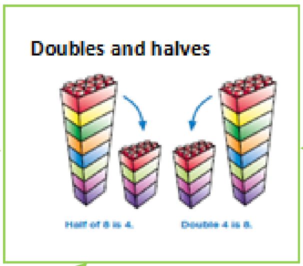
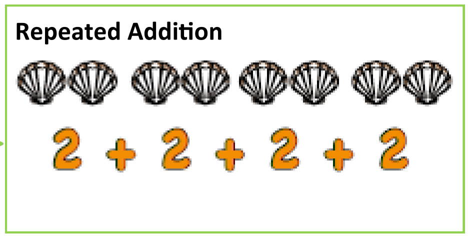
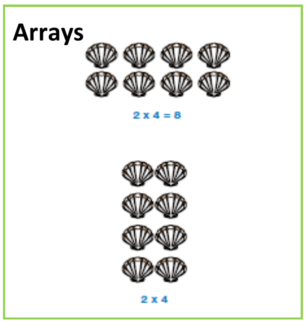
Multiplication and Division

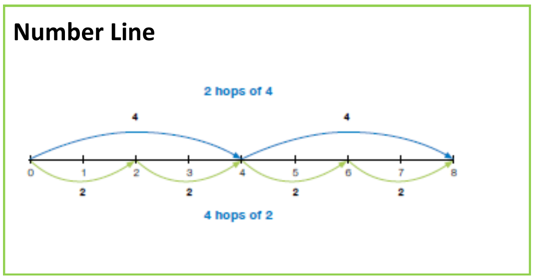
Building blocks...











Leading to...

84 ÷ 7 might be:

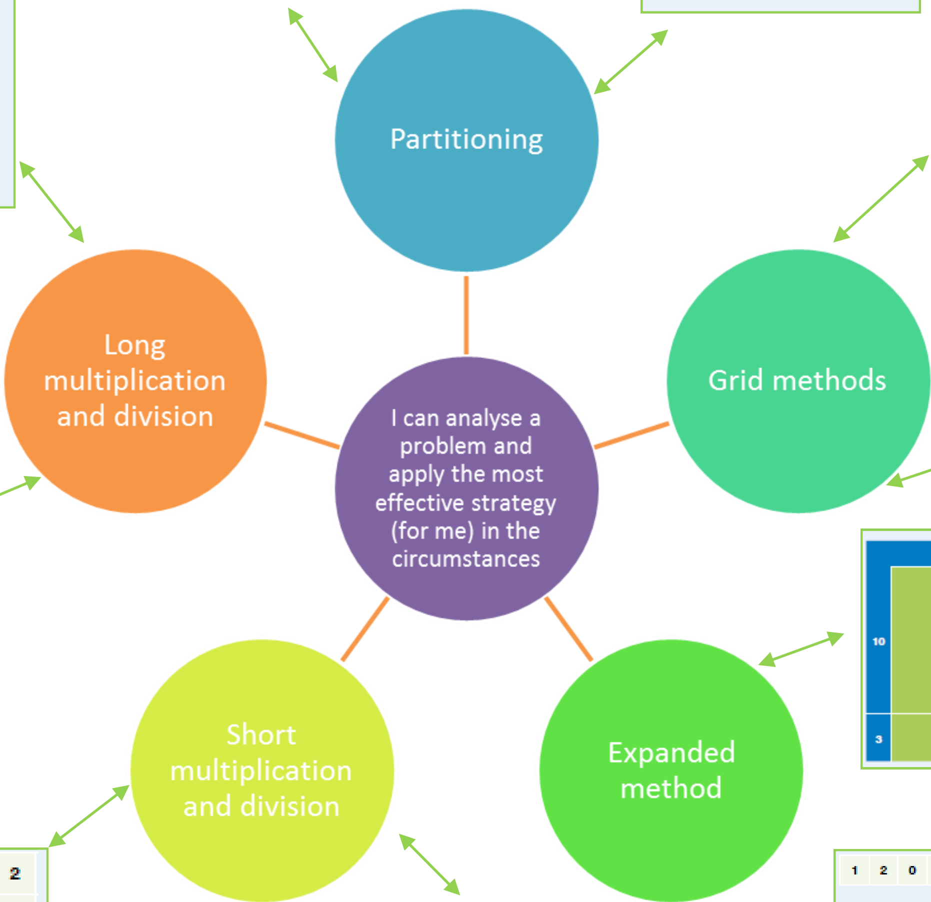
	84			
70	+	14		
			÷	7
10	+	2	=	12

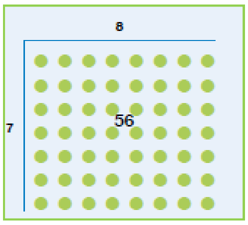
Partitioning: 43 x 6

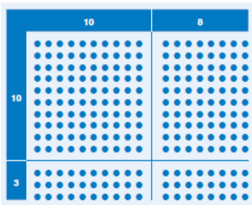
	43			
40	+	3		
			x	6
240	+	18	=	258

	23	R8		
24	560			
–	480	24	x	20
	80			
–	72	24	x	3
	8			

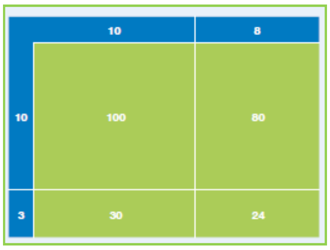
24 x 20







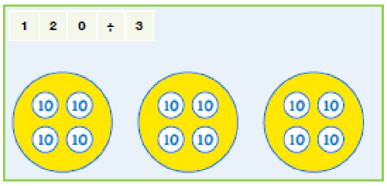
		2	8	6
		x	2	9
			5	4
		7	2	0
	1	8	0	0
		1	2	
	1	6	0	
+	4	0	0	
	8	2	9	4
	2			



	2	4
x		6
1	4	4
	2	

	3	4	2
x			7
1	3	9	4
	2	1	

	40
3	120



Fractions, Decimals and Percentages

The concepts of fractions, decimals and percentages are traditionally considered hard for children and adults to understand. Analysis of SSLN acknowledges that questions relating to fractions, decimals and percentages pose particular problems for children, especially those working at second level. It is well known that children and adults often do not appreciate that fractions, decimal and percentages are equivalent ways of writing the same quantity and that they are different ways of expressing related ideas. It is important that a variety of concrete materials are used to ensure conceptual development of this area.

Fractions

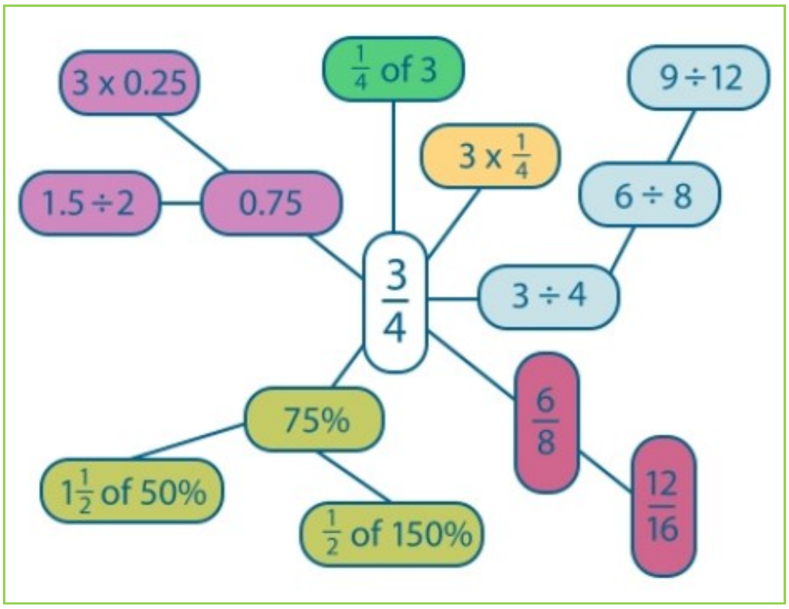
Fractions are numbers, they have a size and a position on a number line. Pupils should be taught how to find fractions of amounts, shapes and numbers. Understanding equivalence is paramount to the understanding of fractions as numbers. Opportunities to link fractions and division should be explored throughout every level.

Decimals

Resources such as Numicon and ten frames are useful in supporting conceptual understanding of decimals. Pupils should be encouraged to make the link between fractions and decimals through exploration using resources that represent fractions that they are familiar with.

Percentages

Many of the difficulties that children (and adults) have when calculating or working with percentages involve seeing figures as whole numbers, as opposed to parts of a whole, and the multiplicative nature of percentages. Therefore it is essential that explicit links be made between known fractions and decimals, using concrete resources, when fractions are being taught.



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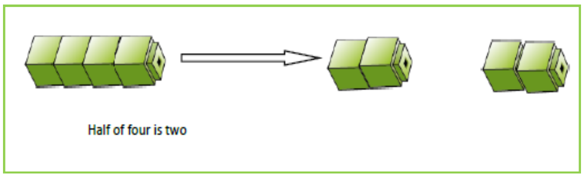
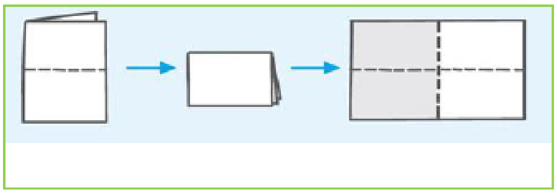
1 http://ken-carroll.com/2007/12/13/linear-and-non-linear-learning/ ; Ken Caroll, December 2007

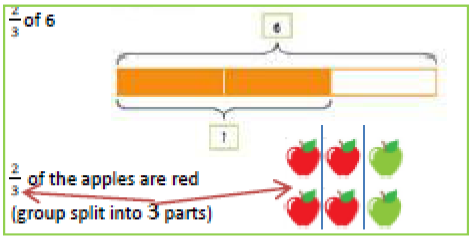
Fractions, Decimals and Percentages

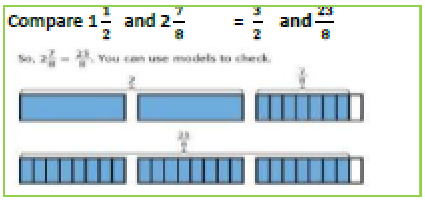
Building blocks...

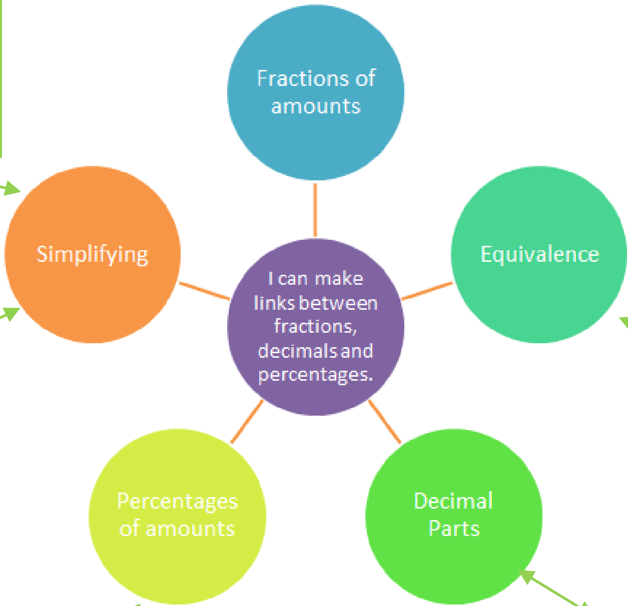


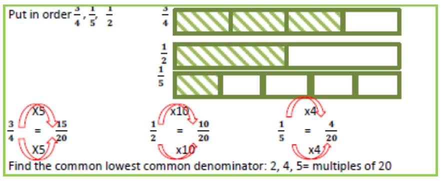


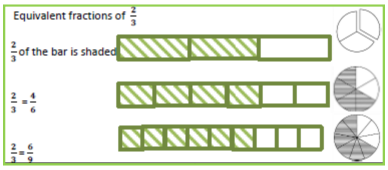
Leading to...

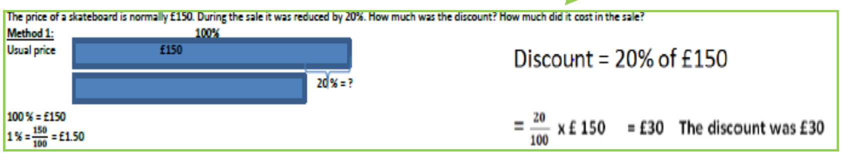


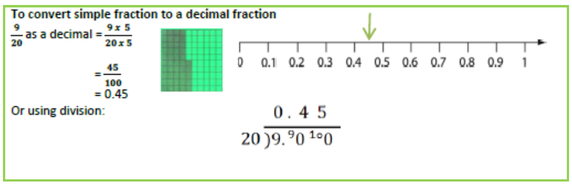












General Principles in Teaching Numeracy and Maths

Think about...

- Early learning of number should focus on oral and mental methods before moving to formal written methods. Moving to formal written methods before understanding is embedded does not effectively support the development of conceptual understanding and number sense.

- Oral mental methods of calculations should be explicitly taught to children and should not exclude them from writing down their working. Mathematical jottings are part of CfE e's and o's.

- When using oral mental methods children should have access to concrete materials and pictorial representations that support them in explaining and recording their thinking.

- Children's understanding should continue to be supported throughout their school experience by the use of the concrete—pictorial—abstract approach.

- Number practice should take place every day allowing children opportunities to explore and extend their understanding of number through talk, think and share.

- Children should be taught times tables with conceptual understanding and through the use of concrete resources and pictorial representations, e.g. counting stick, arrays, counters etc. Discovery and exploration should add value to memorisation of facts and children should be able to use their knowledge of these facts to support further knowledge development e.g. I know 7 x 6 so I can work out 7 x 600.

- Addition and subtraction are the inverse process of each and these skills should be developed together from the earliest age e.g. reciting forward and backward number sequences.

- Multiplication and division are the inverse process of each other and these skills should be developed together e.g. 5 jumps of 4 is 20 so 20 shared by 4 is 5. By developing this understanding our pupils will be able to add subtract, multiply and divide in a variety of ways and not necessarily in the standard written form.

- Problem solving and reasoning should be the crux of every mathematics lesson.

- 'Maths talk' should be evident in our classrooms—children should be able to explain their thinking and strategies used to solve problems.